Rowing Paddles

To what extent has the surface area of rowing paddles been changed over the last half century?

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Introduction

Aim: To what extent has the surface area of rowing paddles been changed over the last half-century?

In order to narrow down this math IA, only two rowing paddles will be analyzed and compared. Both are manufactured and sold by Concept2 as of 2021 (concept2). They will be compared for how much surface area they have and where it is in relation to the shaft of the blade. The first blade will be the iconic Macon blade that dominated the competitive rowing scene in the 1960s (gvsu.edu). They are rarely used nowadays due to more efficient asymmetric blades, Concept2 still sells them as somewhat of a novelty item for traditionalists. The second blade will be the much newer Fat2 that was released in 2006 (concept2) and it is also the most primary blade model that my rowing club uses.

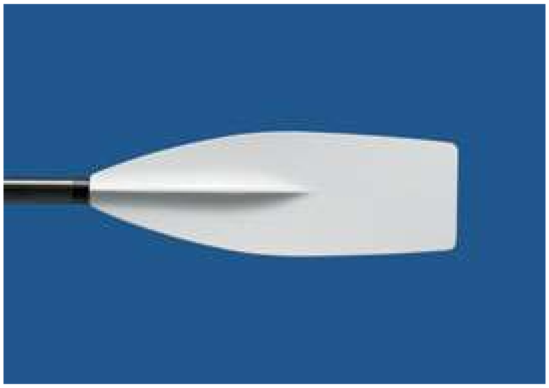


Figure 1: Macon Blade from the concept2 website

Figure 2: Macon Blade from the concept2 website

Something to make clear for the context of this IA is that both blades compared will be of the type used for sculling instead of sweeping. The difference is that each person needs two sculling blades to row, whereas sweepers only need one much larger blade. They are essentially different categories of rowing that compete in races separately from one another. I am choosing to analyze the sculling blade as it's the primary style of rowing my club focuses on and it's also what I myself am more familiar with.

It should also be noted that a concave dip in the face of either blade will not be considered in any of the calculations. As they are far beyond what I am able to learn within the span of the IA process. As an example of what is being referred to, images are given below.

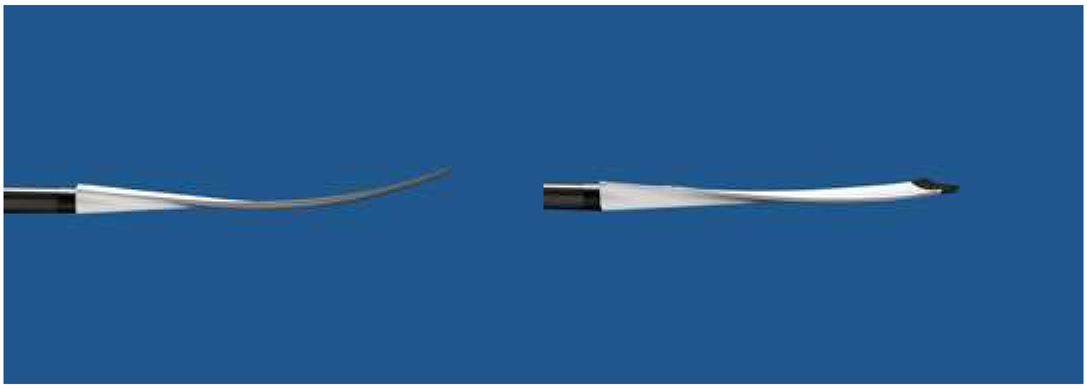


Figure 3: Side view of Macon and Fat2 blade from concept2 website.

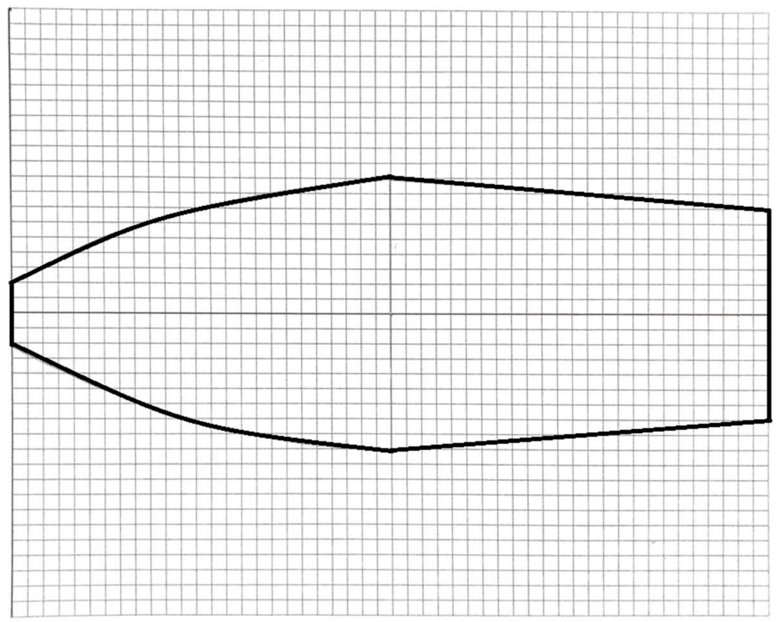
The Macon blade used is the medium scull as opposed to the large scull. The different sizes only differ by a single centimetre at certain locations. Lastly, are no size options for Fat2 sculling blades.

Approaching the Problem

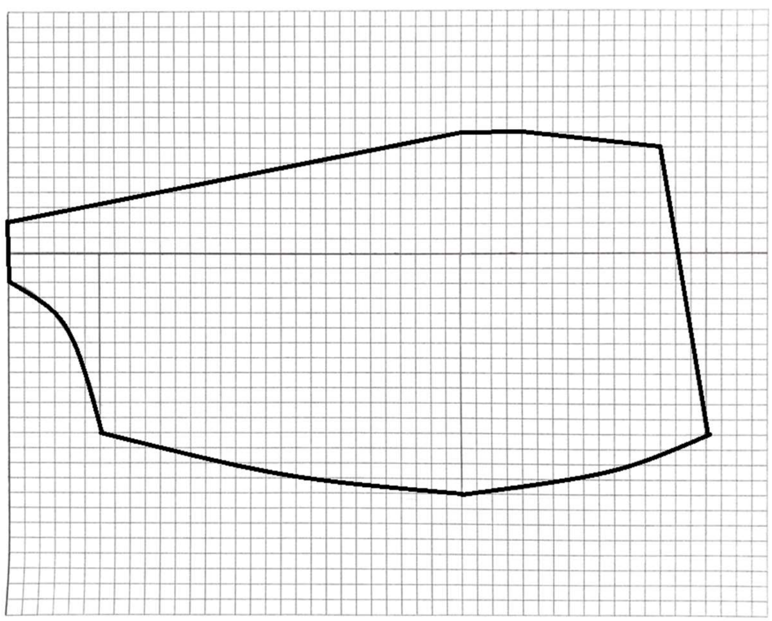
When initially approaching this investigation, my first instinct was to overlay a transparent graph over the images of the two different blades. However, due to a myriad of technical difficulties, I resorted to drawing them by hand on a piece of 40×50 piece of graph paper. Each square as the side length of $0.5cm$ and was also equivalent to $1cm$ of the actual oar. Thus, the scale of both drawings should be 1:2 if one were to print them out on a standard sheet of A4 paper. Hand drawing also had a few minor issues, the major one being the slight inaccuracy of certain aspects of the paddle. However, it is still precise enough to calculate the different areas as the primary dimensions are all met. Below is a table of all the dimensions supplied on the official Concept2 website.

	Medium Macon Sculling Blade	Fat2 Sculling Blade
Blade Length	$50cm$	$46cm$
Width at Tip	$14cm$	$16.5cm$
Width at Broadest Point	$17cm$	$23cm$

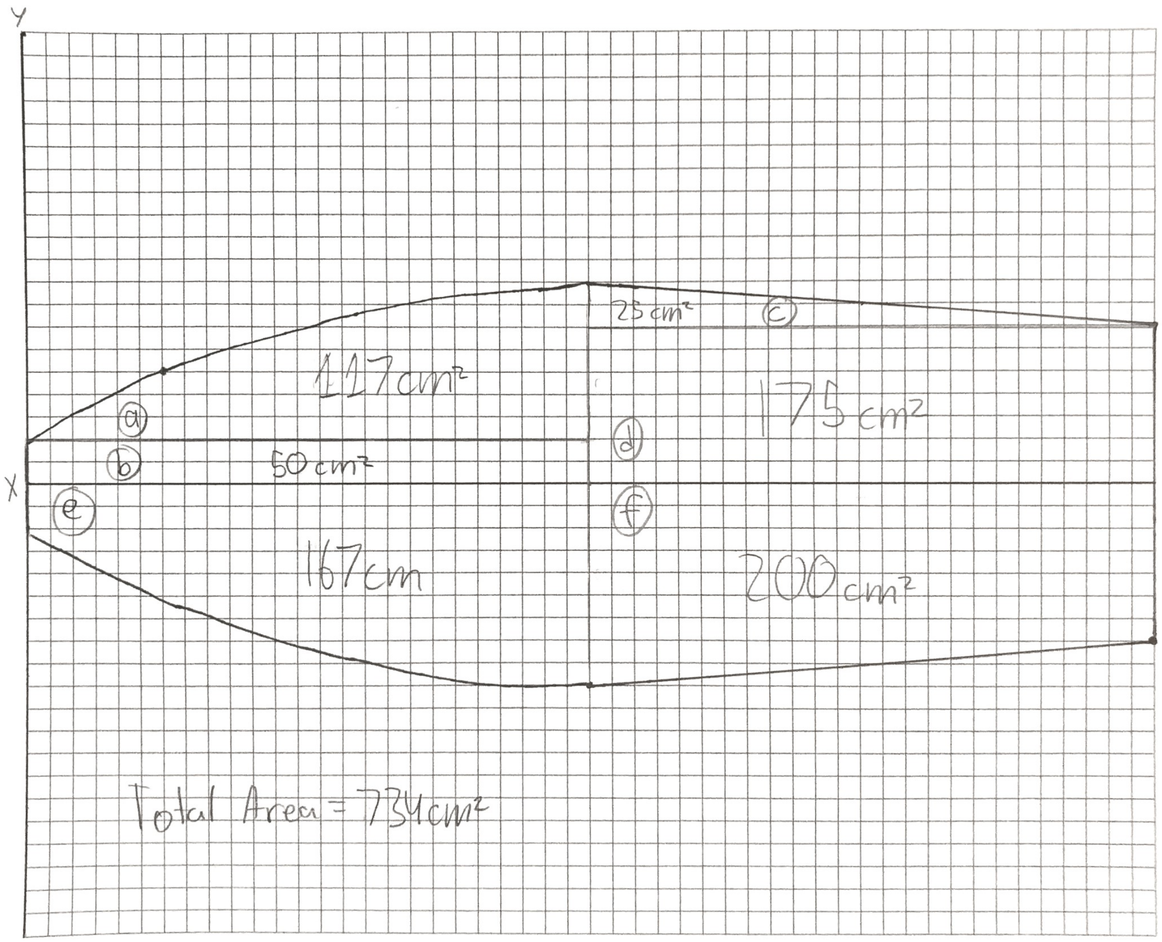
It should be noted that the long horizontal line extrapolates where the carbon fibre shaft of the blade would reside, and the vertical line represents the "width at broadest point" measurement in the chart above.



Before solving for the total areas of either blade, they were divided up into as many simple polygons as possible in order to isolate the areas that required solving for the equations and integrals of parabolas. Also, on the note of parabolas, the orientation does not matter as I am only seeking either the area above or below the curve. As a product of the previous statement, all areas must be positive as it sometimes may be easier to solve for an area when the parabola equation is given in a manner that puts the needed area below the x-axis.



Macon Medium Scull Blade



Area a

$$x = a(y - 7)(y + 7)$$

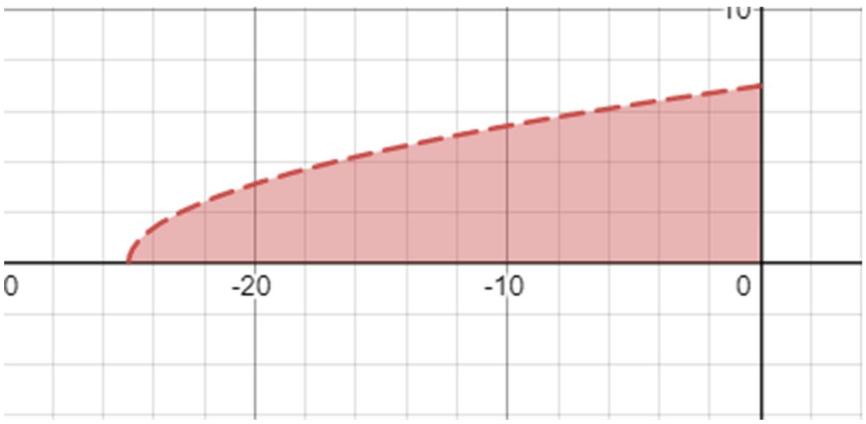
$$-25 = a(-7)(7)$$

$$-25 = -49a$$

$$a = \frac{25}{49}$$

$$x = \frac{25}{49}(y-7)(y+7)$$

$$x = \frac{25y^2}{49} - 25$$



$$A = \int_0^7 \frac{25y^2}{49} - 25)\,dy$$

$$A = \left[\frac{25 \times y^3}{49 \times 3} - 25y\right]_0^7$$

$$A = \frac{25 \times 7^3}{49 \times 3} - (25 \times 7)$$

$$A = \frac{-350}{3} \approx -116.\bar{6}$$

However, this sector must have a positive area when calculating for the total area of the paddle and it also must be rounded down to 3 sig figs. Thus, the area used is in the final calculations is $A = 117cm^2$.

Area b

$$A = 2 \times 25 = 50cm^2$$

Area c

$$A = \frac{2 \times 25}{2} = 25cm^2$$

Area d

$$A = 7 \times 25 = 175cm^2$$

Area e

This area is equivalent to the sum of areas a and b

$$A = 117 + 50 = 50cm^2$$

Area f

This area is equivalent to the sum of areas c and d

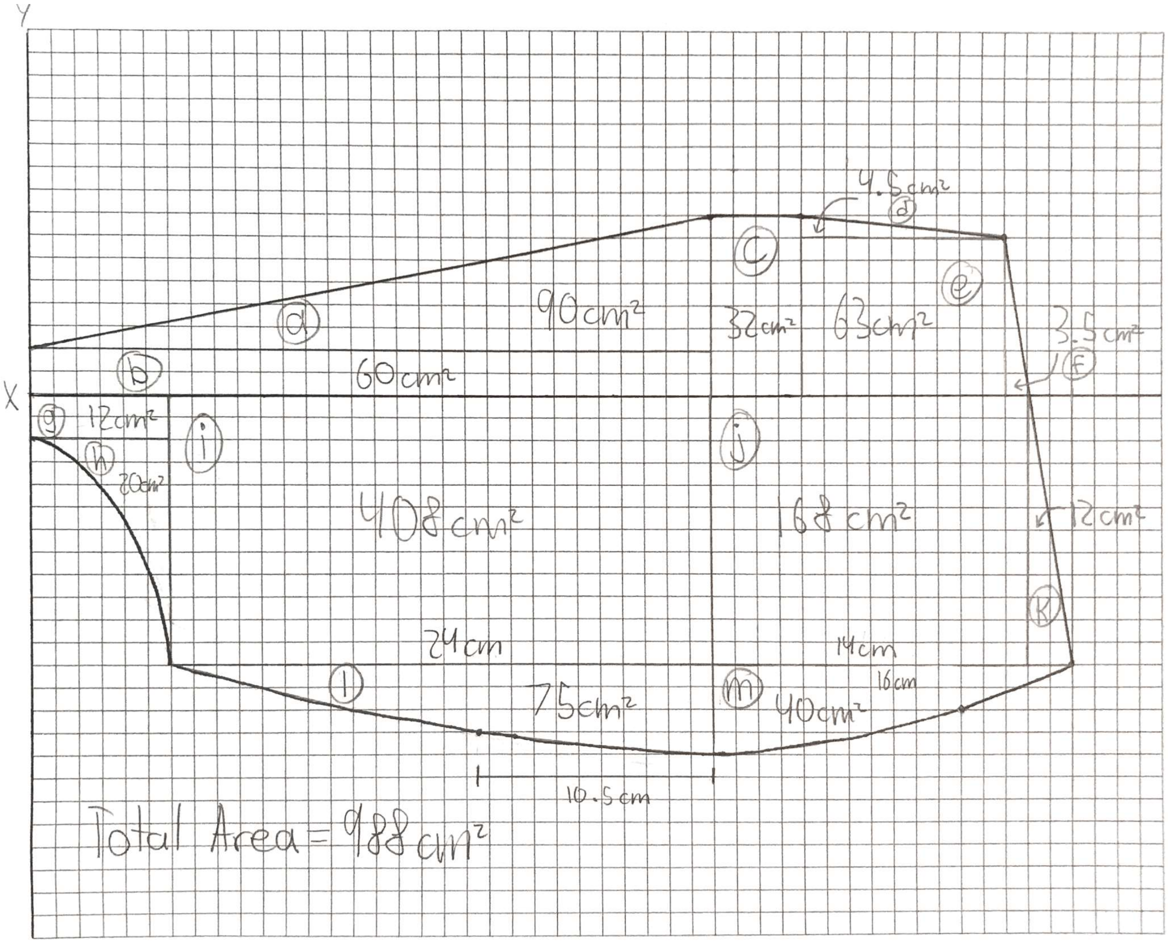
$$A = 25 + 200 = 50cm^2$$

Total area

The sum of all areas listed in this section

$$A = 117 + 50 + 25 + 175 + 167 + 200 = 734cm^2$$

Fat2 Scull Blade



Area a

$$A = \frac{6 \times 30}{2} = 90cm^2$$

Area b

$$A = 2 \times 30 = 60cm^2$$

Area c

$$A = 4 \times 8 = 32cm^2$$

Area d

$$A = \frac{1 \times 9}{2} = 4.5cm^2$$

Area e

$$A = 7 \times 9 = 63cm^2$$

Area f

$$A = \frac{1 \times 7}{2} = 3.5cm^2$$

Area g

$$A = 2 \times 6 = 12cm^2$$

Area h

Vertex (h, k) parabola form equation: $y = a(-h + x)^2 + k$

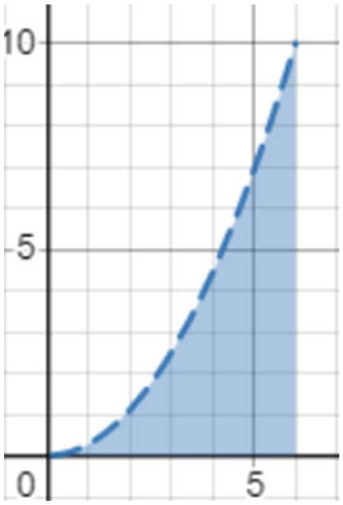
Parabola passes though point (6,0) and vertex (0,10)

$$y = ax^2 + 10$$

Plugging in the point to solve for a: $0 = 36a + 10$

$$a = -\frac{5}{18}$$

$$y = 10 - \frac{5x^2}{18}$$



The correct parabola function is found but when looking for the area under the curve rather than inside the curve, this modified parabola with the same curve is much easier to work with.

$$y = \frac{5x^2}{18}$$

$$A = \int_0^6 \frac{5x^2}{18} dx$$

$$A = \left[\frac{5 \times 6^3}{18 \times 3}\right]_0^6$$

$$A = 20cm^2$$

Area i

$$A = 14 \times 24 = 408cm^2$$

Area j

$$A = 14 \times 14 = 168cm^2$$

Area k

$$A = \frac{2 \times 12}{2} = 12cm^2$$

Area l

General parabola equation: $y = ax^2 + bx + c$

Parabola passes though points: $[(4,0), (0, -34), (3, -10.5)]$

Point $(0, -34)$: $-34 = 0a + 0b + c$

$c = -34$

Point $(4,0)$: $0 = 16a + 4b + c$

Point $(3, -10.5)$: $-10.5 = 9a + 3b + c$

Inputting $c = -34$:

$$0 = 16a + 4b - 34$$

$$34 = 16a + 4b$$

$$34 = 4(4a + b)$$

$$\frac{17}{2} = 4a + b$$

$$b = \frac{17}{2} - 4a$$

$$-10.5 = 9a + 3b - 34$$

$$23.5 = 9a + 3b$$

$$23.5 = 3(3a + b)$$

$$\frac{47}{6} = 3a + b$$

Solving for a:

$$\frac{47}{6} = 3a + (\frac{17}{2} - 4a)$$

$$-\frac{2}{3} = -a$$

$$a = \frac{2}{3}$$

Solving for b:

$$-10.5 = 9a + 3b + c$$

$$-10.5 = \frac{9 \times 2}{3} + 3b - 34$$

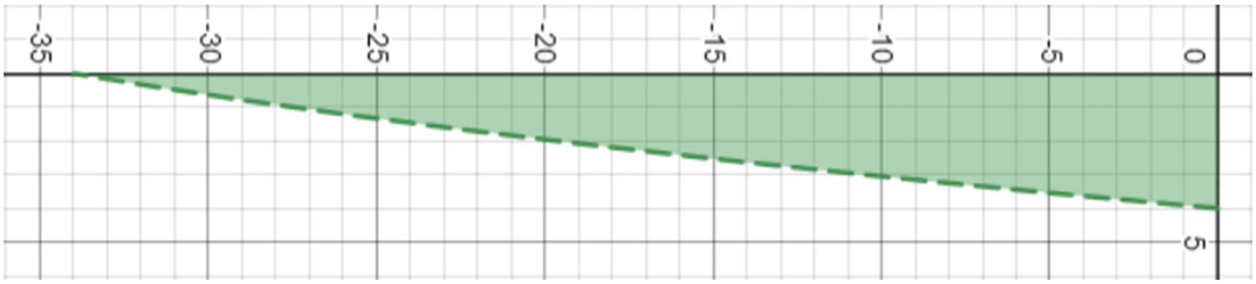
$$23.5 = 6 + 3b$$

$$17.5 = 3b$$

$$b = \frac{35}{6}$$

Final equation of the parabola:

$$y = \frac{2x^2}{3} + \frac{35x}{6} - 34$$



Solving area

$$A = \int_0^4 \left(\frac{2x^2}{3} + \frac{35x}{6} - 34\right) dx$$

$$A = \left[\frac{2x^3}{9} + \frac{35x^2}{12} - 34x\right]_0^4$$

$$A = \frac{2 \times 4^3}{9} + \frac{35 \times 4^2}{12} - 34 \times 4$$

$$A = -75.\overline{1}cm^2$$

However, this sector must have a positive area when calculating for the total area of the paddle and it also must be rounded down to 2 sig figs. Thus, the area used is in the final calculations is $A = 75cm^2$.

Area m

General parabola equation: $y = ax^2 + bx + c$

Parabola passes though points: $[(0, -16), (4,0), (2, -11)]$

Point $(0, -16)$: $-16 = 0a + 0b + c$

$c = -16$

Point $(4,0)$: $0 = 16a + 4b + c$

Point $(2, -11)$: $-11 = 4a + 2b + c$

Inputting $c = -16$:

$$0 = 16a + 4b - 16$$

$$16 = 16a + 4b$$

$$16 = 4(4a + b)$$

$$4 = 4a + b$$

$$b = 4 - 4a$$

$$-11 = 4a + 2b - 16$$

$$5 = 4a + 2b$$

$$5 = 2(2a + b)$$

$$2.5 = 2a + b$$

Solving for a:

$$2.5 = 2a + (4 - 4a)$$

$$-1.5 = -2a$$

$$a = \frac{-1.5}{-2} = \frac{3}{4}$$

Solving for b:

$$-11 = 4a + 2b + c$$

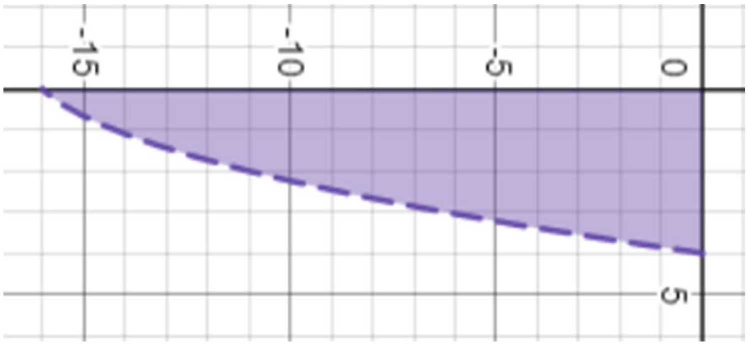
$$-11 = \frac{4 \times 3}{4} + 2b - 16$$

$$5 = 3 + 2b$$

$$b = 1$$

Final equation of the parabola:

$$y = \frac{3x^2}{4} + x - 16$$



Solving area

$$A = \int_{0}^{4} \left(\frac{3x^2}{4} + x - 16 \right) dx$$

$$A = \left[\frac{x^3}{4} + \frac{x^2}{2} - 16x \right]_{0}^{4}$$

$$A = \frac{4^3}{4} + \frac{4^2}{2} - 16 \times 4$$

$$A = -40cm^2$$

Similarly, to before, the area must be positive as it would not make sense to have negative area when combining the sum of all areas.

Total area

The sum of all areas listed in this section

$$A = 90 + 60 + 32 + 4.5 + 63 + 3.5 + 12 + 20 + 408 + 168 + 12 + 75 + 40 = 988cm^2$$

Area above and below the shaft

Area above the shaft

$$A = 90 + 60 + 32 + 4.5 + 63 + 3.5 = 253cm^2$$

Area below the shaft

$$A = 12 + 20 + 408 + 168 + 12 + 75 + 40 = 735cm^2$$

Conclusion

To answer my research question about the extent that the surface areas of rowing paddles have changed over the last half century, it is firstly obvious that the modern Fat2 blade has a much higher surface area ($988cm^2$) as opposed to $734cm^2$. Nearly a 26% increase in total surface area. Additionally, the majority of the blade face is now below the shaft whereas in the past, the Macon blade had a symmetrical edge above and below. Nowadays, the Fat2 blade has over 65% of the blade under the shaft. This suggests either a massive shift in the rowing technique or someone realized that it was inefficient to put so much of the blade above the water where it would not provide as much resistance to the water. This would begin to transition to a physics investigation rather than a purely mathematical one.

This mathematical investigation has allowed me to further pursue my passion of rowing from both a mathematical and slightly historical perspective. If this investigation was repeated, I would consider an improvement by having all the parabola equations stem from the same starting point as this would both demonstrate a higher understanding of mathematics and also makes everything neater and more presentable. A limitation that I have considered is potential for error due to the graph being hand drawn rather than digitally created. I have tried for hours but to no avail in attempting to digitally overlay a grid over the image of the paddles in adobe photoshop. Drawing it by hand still proved to be difficult and required multiple attempts before achieving a successful outline of the oar. Another limitation would be the fact that the faces of the blades are curved. This was mentioned and presented earlier in the IA in figure 3. The implications of this would be that the final area that is solved for mathematically is not fully accurate as it does not account for the convex curve of the face of the blade. The curve is much more prominent in the newer fat2 blade but is still present in the older macon blade. If I were to repeat this IA with a further understanding of mathematics a few years down the line, this would be the number one thing I would consider adding.

There are few implications of the data that was gathered and analyzed from this IA as the numbers can only mean so much. To take this idea a step further, I would consider doing a physics IA based on this topic where I would calculate how the difference in area and where the area is affects the overall efficiency and effectiveness of the two blades. Otherwise, there is little real world implications that can be concluded from the results of this investigation.

Finally, through the process of this IA, I was able to better understand and apply my knowledge of calculus in finding the equations and eventually the areas under parabolas. Additionally, I am now confident in my skills to accurately and effectively convey mathematical ideas through word with formatting the equations.

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